

Challenge Question

SIMC 2010

1 Story

Coffee Bean and Starbucks are competing café networks or chains. You are a Coffee Bean Area Representative in Singapore. Your job is to open new café(s) in your area with the goal to attract more customers in the presence of your competitor.

2 Simple Model

For simplicity, we assume that potential customers always choose a café closest to where they are currently on the streets. Thus the main idea is to position your coffee café closer to potential customers than Starbucks café.

We may also assume that potential customers are distributed evenly along the streets and the area population is sufficiently large to be considered continuous. In other words, the number of potential customers along any street segment is proportional to the length of the segment. Thus we can ignore the actual number of potential customers and instead find the ratio of Coffee Bean customers to Starbucks ones.

A schematic street map with lengths of streets and locations of present cafés (Coffee Bean and Starbucks cafés) indicated is given (see for example, Figure 5). Width and actual shape of streets are neglected as well as locations and size of buildings, traffic, landscape etc. We assume that the length of all streets are **integers** and that you can only open new café at **integer points** i.e. locations marked with • in the schematic in Figure 5. You are **not allowed** to open on the same spot as the existing cafés.

For each point of each street, we find the café closest to it, that is, whose distance along the streets is the shortest. If a point is closer to a Starbucks café, a potential customer at that point will choose Starbucks. Similarly, if a point is closer to a Coffee Bean café; a potential customer at that point will choose Coffee Bean. However, if a point is equidistant from Starbucks or Coffee Bean cafés, a potential customer at that point will choose either Starbucks or Coffee Bean equally likely, regardless of the number of cafés from each network or chain that it is closest to. Therefore, every point is controlled either by Starbucks, Coffee Bean or doubly-controlled.

Let C , S and D be segments controlled by Coffee Bean, Starbucks, or double respectively and let $l(C)$, $l(S)$ and $l(D)$ be the total lengths of streets belonging to each of the segments. Since D is equally shared by the two networks or chains, the total share of Coffee Bean customers is

$$\frac{l(C) + \frac{1}{2}l(D)}{l(C) + l(S) + l(D)},$$

where $l(C) + l(S) + l(D)$ is the total length of all streets in our area.

Example 1

The map of the area is shown in *Figure 1*. There are only 4 streets here, each of length 1 and only 1 junction. The locations of 2 Starbucks cafés are given by square dots and a Coffee Bean one by a circle dot. The segment controlled by Coffee Bean is highlighted with red, the Starbucks one is cyan and the double-controlled segment is blue. As you can see from the illustration, we have

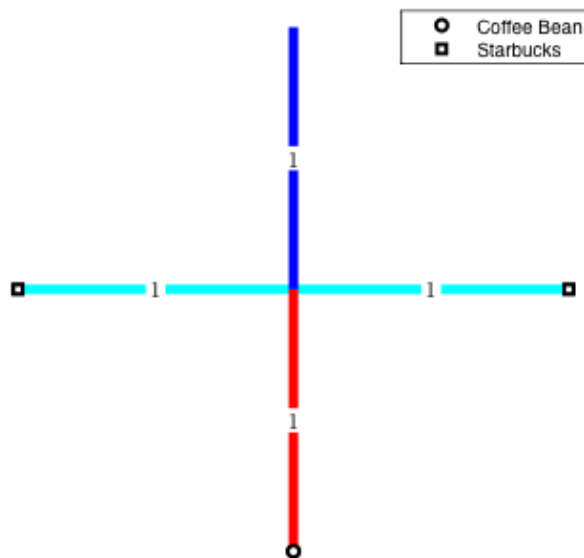


Figure 1. The map with streets, café locations, and distances indicated

$$l(C) = 1, \quad l(S) = 2, \quad l(D) = 1, \quad l(C) + l(S) + l(D) = 4$$

Thus the share of Coffee Bean customers is $\frac{l(C) + \frac{1}{2}l(D)}{l(C) + l(S) + l(D)} = \frac{1 + \frac{1}{2}}{4} = \frac{3}{8}$ and Starbucks is $\frac{5}{8}$, so Starbucks wins the competition.

Example 2

Suppose now that initially Coffee Bean loses to Starbucks but we are given money to open some new cafés in order to attract more customers.

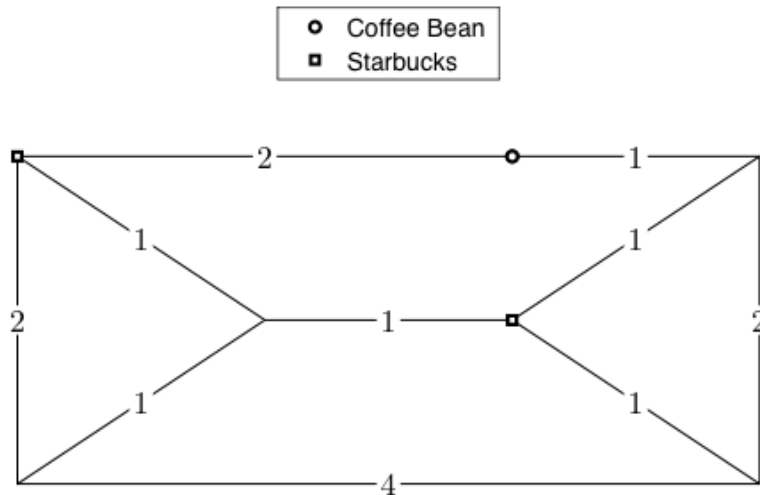


Figure 2a: The map with streets, café locations, and distances indicated

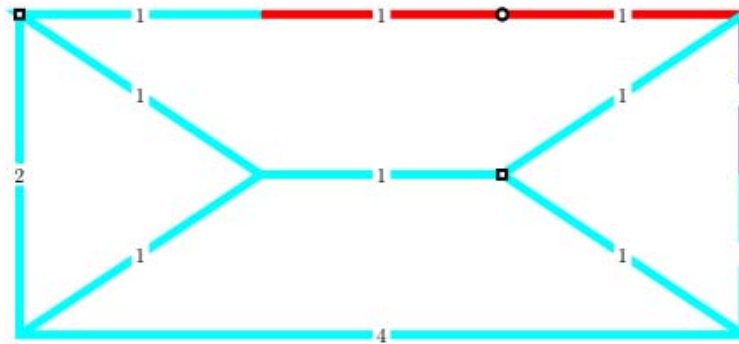


Figure 2b: Segments controlled by Coffee Bean (red), Starbucks (cyan), and double-controlled (blue).

The map of the area with locations of existing cafés is shown in *Figure 2a*. All distances between cafés and street junctions are also indicated. The total length of all streets here is $1 + 1 + 1 + 1 + 1 + 1 + 1 + 2 + 2 + 2 + 4 = 16$.

Further, as *Figure 2b* shows, we have $l(C) = 2$, $l(D) = 1$, $l(S) = 13$ and hence the share of customers attracted by Coffee Bean is $\frac{2.5}{16}$ while Starbucks is $\frac{13.5}{16}$.

However, if Coffee Bean opens two new cafés as shown in *Figure 3*, then we have $l(C) = 8$, $l(D) = 2$, $l(S) = 6$ and the share of customers attracted by Coffee Bean is $\frac{9}{16}$, which means that Coffee Bean wins the competition.

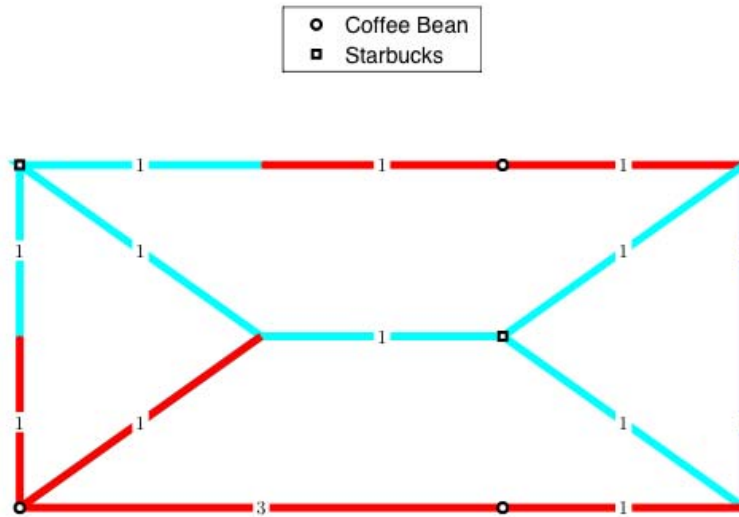
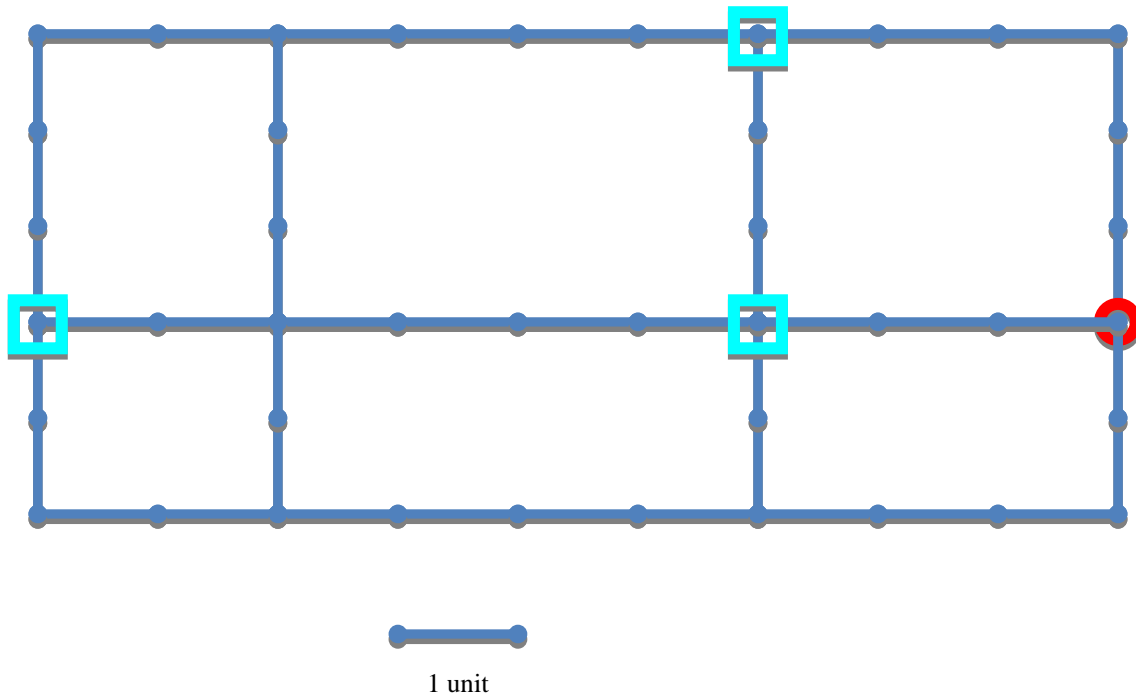


Figure 3: Map with two new Coffee Bean cafés.

CHALLENGE 1 (Simple Model)

Your goal is to win the business competition (that is, attract more customers than Starbucks) by opening *as few new cafés as possible*. You are to:

- Outline your strategy of locating new cafés for a general map and explain why it, indeed, produces *as few new cafés as possible* sufficient to win.
- Apply your strategy to the schematic given below (Starbucks in cyan and Coffee Bean in red). Explain your method clearly.



- Apply your strategy to the given map of City Hall area in *Figures 4 and 5*. Explain your method clearly.

Your work will be judged based on

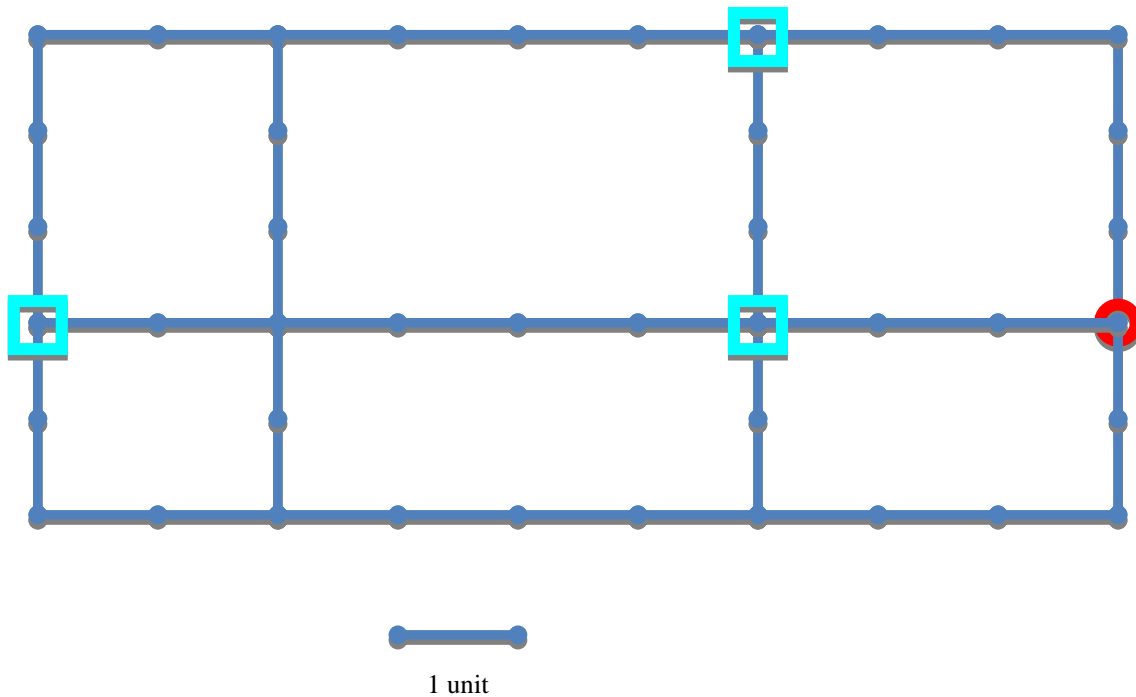
- Optimality of your solution (the number of cafés must be as small as possible),
- Justification of your method (you should explain why your method will help you achieve the objective),
- Application of your strategy to the specific examples.

CHALLENGE 2 (Model with Budget)

In reality there are at least two other factors that have to be considered. First, the cost of opening a new shop depends on the location and there is a finite budget. Second, we are interested not in winning the competition but in *attracting as many customers as possible*.

Assume that the cost of opening up a new café is k units at a k -junction. In particular, it is 1 unit at a dead end, 2 units along a road, 3 units at a T-junction etc. You are to:

- Outline your strategy of locating new cafés for a general map with a budget of B units, and explain why it, indeed, maximizes $l(C) + \frac{1}{2}l(D)$ where C is the segments controlled by Coffee Bean and D doubly-controlled?
- Apply your strategy to the schematic given below (Starbucks in cyan and Coffee Bean in red) with $B = 7$. Explain your method clearly.



- Apply your strategy to the given map of City Hall area in *Figures 4 and 5* with $B = 13$. Explain your method clearly.

Your work will be judged based on

- Optimality of your solution (the share of Coffee Bean customers should be as big as possible),
- Justification of your method (you should explain why your method gives the largest share of customers),
- Application of your strategy to the specific examples.

CHALLENGE 3 (Generalised Model)

The first two models are very much simplifications of the real situation. In reality, customers might not be evenly distributed among the streets and the behaviour of the potential customers may be influenced by factors other than just distance alone. Also, there are limited places where new cafés can be opened.

Propose a few more general models that you can think of and describe their strengths and weaknesses. Think of real life scenarios that might be included in your models, such as pedestrian density, rental cost, availability of space, weather conditions, the size of budget etc that you can think of. You need to explain how your model takes into account these scenarios, state meaningful objectives to be optimised and outline your approach to solve the problem.

Your work will be judged based on

- Formulation and justification of your models;
- Solvability of your models and critical analysis of your solution.

Map of City Hall Area

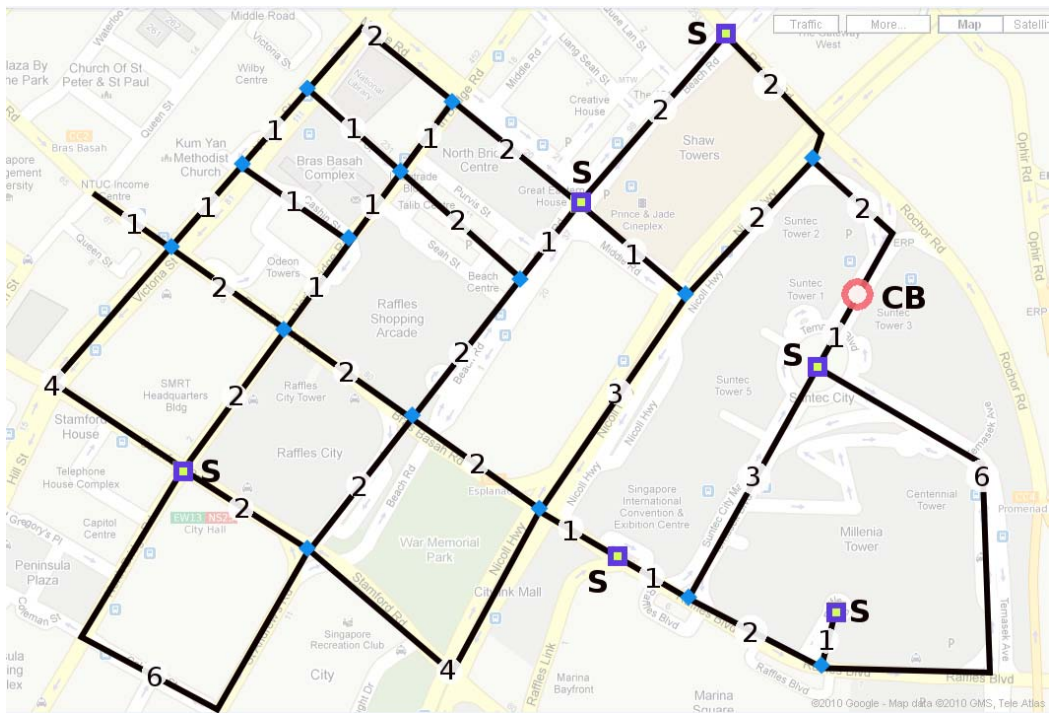


Figure 4: Map of the City Hall area: **S** — Starbucks; **CB** — Coffee Bean; small squares — junctions, numbers indicate distances between junctions and coffee shops.

Starbucks in cyan and Coffee Bean in red

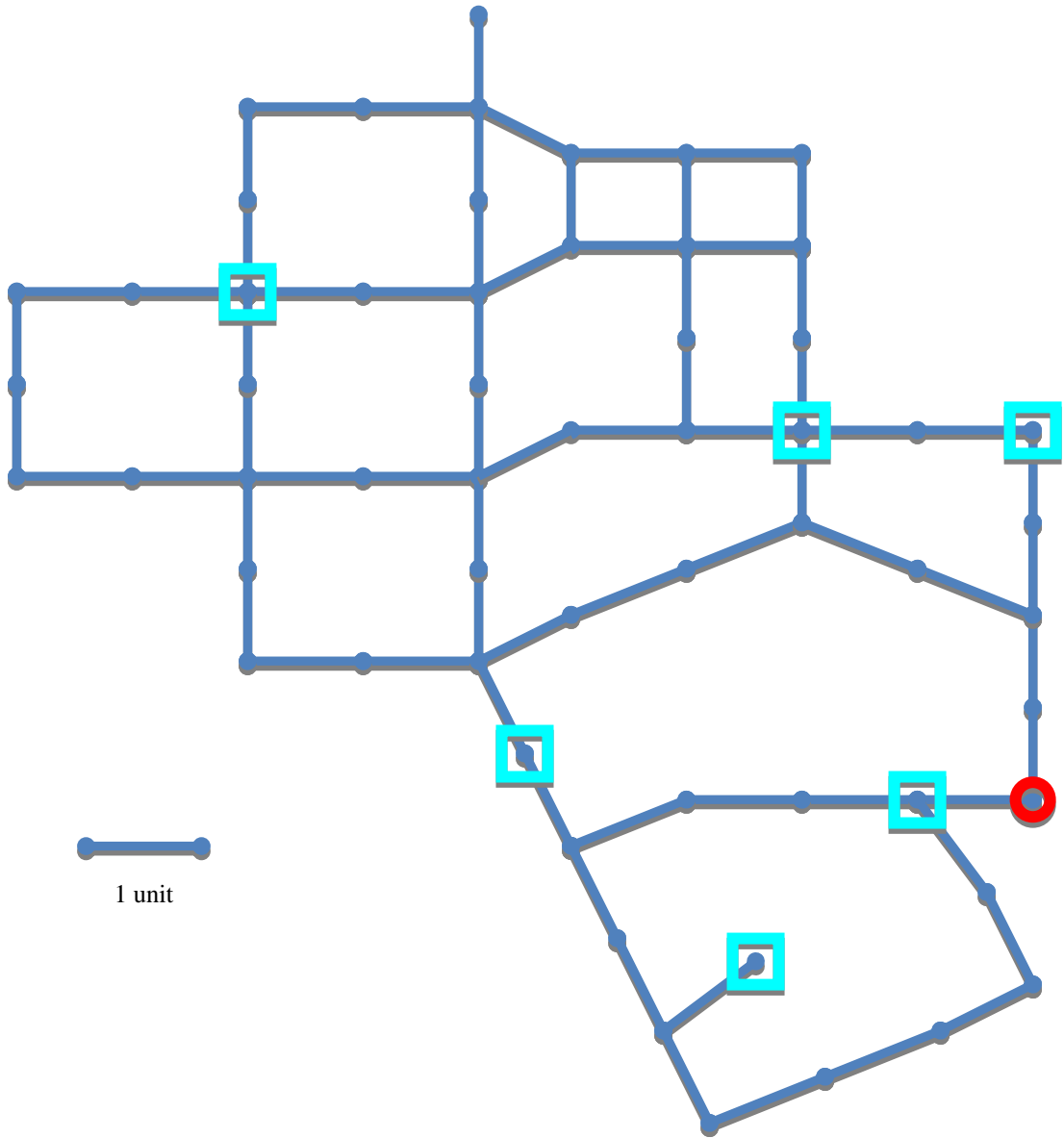


Figure 5. Schematic of City Hall area