Singapore International Mathematics Challenge 2008

On-Site Challenge

Over the last four decades, containerization has revolutionized cargo shipping worldwide. Containers are large boxes of standard dimensions used to carry different types of cargo. Containers possess several advantages: they require less product packaging, they help reduce damage and yield higher productivity during the various handling phases. Most of the shipping cargo is now transported using containers that are stacked on ships. A container terminal in a seaport is the place where ships dock on berths. At the berthing location, quay cranes are used to unload/load containers from/to the ship. The unloaded containers are transported by vehicles to a buffer area called the yard where they are stored temporarily before being picked up by the customer for whom it is destined. Likewise the containers to be loaded are moved from the yard to the ship which will carry it to another destination port in the world. Figure 1 provides a schematic of a typical port layout.

With increasing competition among different terminals, it is critical for a container terminal to reduce the waiting time that ships incur at the port before being berthed. Once a ship is berthed it will remain in location until all the required container processing is done. At the beginning of each planning period, an expected arrival time is estimated for all the ships arriving at the port over that period. A berth can be viewed as a linear stretch in the port with the length that each ship occupies along the berth known precisely. The expected port stay time for each ship for performing the unloading and loading operations is also estimated beforehand. Due to space restrictions at the berth side and multiple ships arriving around the same time, some ships might be forced to wait at the port before being berthed. A berth allocation plan can be modelled on a rectangular plane with space (berth) along the Y-axis and time along the X-axis. Each ship in this representation is a rectangle with ship length as its height and the expected port stay as its width. Clearly, two ships cannot share the same location at any given time. Figure 2 shows a simple space-time representation of a berth allocation plan.

At the start of each planning period, your objective as a port operator is to develop a good berthing plan that determines the optimal location and optimal berthing time for the set of ships arriving over the period such that the waiting times are minimal. A data set is provided in Table 1. An additional issue is that the actual arrival times and the port stay times are often delayed beyond the estimated values. In the planning phase it is important to develop a robust berthing plan that can handle delays without making too many modifications to the proposed plan during the actual implementation phase.

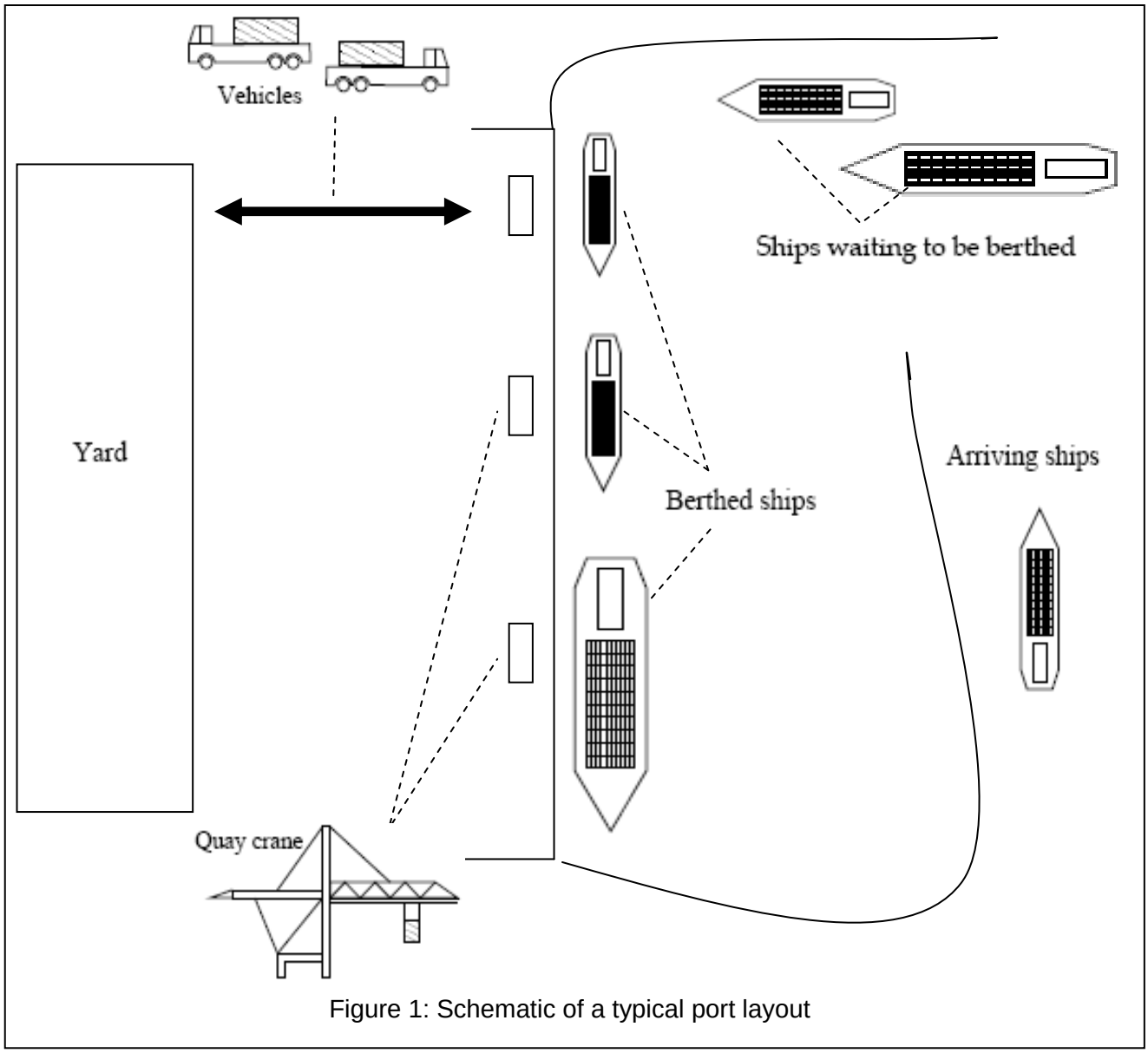

Figure 1: Schematic of a typical port layout

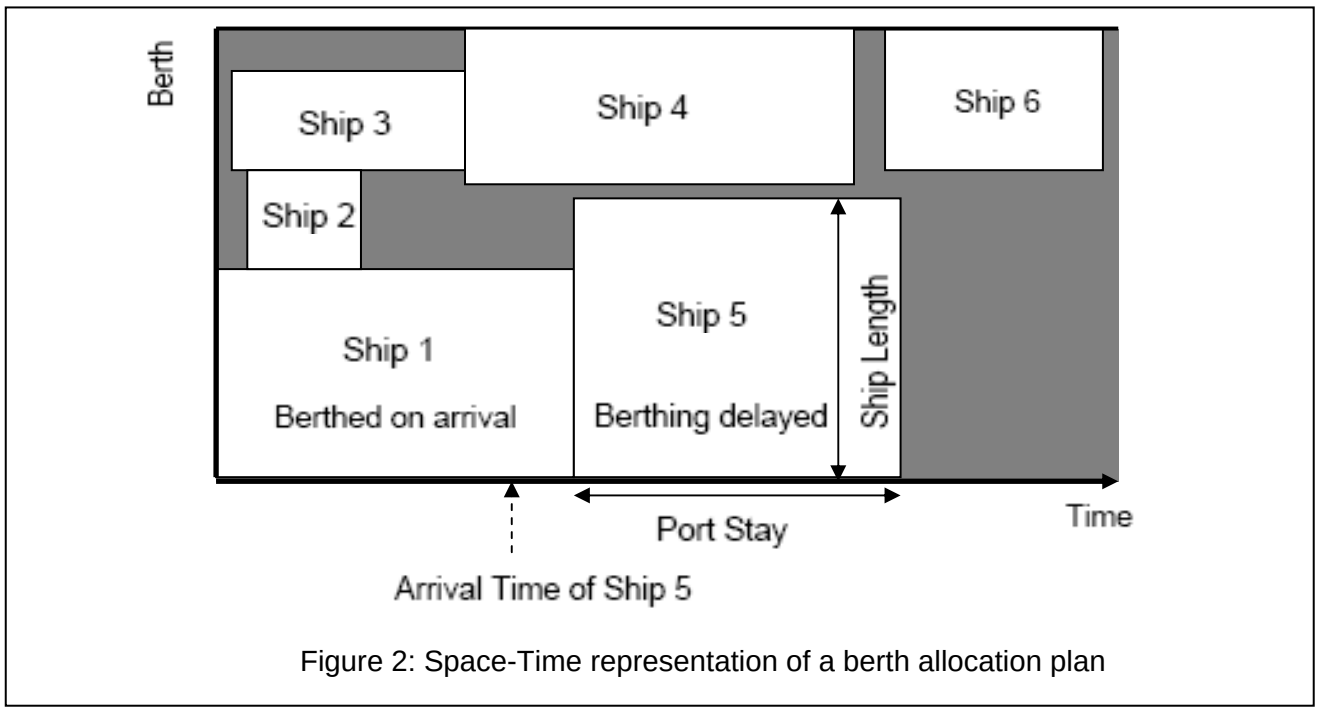

Figure 2: Space-Time representation of a berth allocation plan

Berth Length = 2400 metres

Planning Horizon = Ships arriving over the next five days

Ship_ID	Expected_Arrival_Time(Hours)	Expected_Port_Stay_Time(Hours)	Ship_Length_(Metres)
1	0	17	276
2	1	21	167
3	1	19	208
4	4	40	277
5	5	4	102
6	11	26	168
7	11	6	150
8	12	16	261
9	13	5	102
10	17	22	276
11	18	11	231
12	20	24	207
13	22	18	208
14	27	19	277
15	29	12	231
16	31	2	102
17	34	18	222
18	41	29	300
19	42	8	116
20	42	35	276
21	45	31	340
22	45	35	183
23	47	7	68
24	50	24	294
25	53	18	292
26	55	2	53
27	56	9	91
28	58	4	53
29	58	8	72
30	64	27	275
31	66	10	68
32	70	7	117
33	72	11	183
34	73	15	294
35	74	19	152
36	75	4	118
37	80	25	145
38	80	15	150
39	81	19	147
40	81	5	150
41	82	9	77
42	83	33	190
43	87	8	68
44	89	13	264
45	90	14	147
46	90	12	148
47	92	8	91
48	92	7	148
49	93	11	145
50	96	10	150
51	97	40	232
52	101	9	79
53	101	16	121
54	106	6	79
55	106	5	70
56	107	9	147
57	109	13	205
58	111	28	300
59	113	7	78
60	114	5	124
61	117	9	192
62	119	6	117

Table 1: Dataset for 62 ships over a 5 day planning horizon